Di erential Equations Prelim Spring 2014

May 16, 2014

0.1 ODEs and O Es

1. **Find the equilibria and their stability types for the system**

$$\dot{x} = \alpha x + y$$
$$\dot{y} = x^2 + x - y^2$$

2. **Analyze the following system and show that it has a closed orbit.**

$$\dot{p} = q$$
$$\dot{q} = p + pq - p^3$$

3. **Analyze the system**

$$r_{n+1} = 1 + \alpha s_n - \beta |r_n|$$
$$s_{n+1} = -r_n,$$

especially for the values $\alpha = 0, 1/2, 1$ **and** $\beta = 1, \frac{7}{4}$ **-3.W2(a)-.da6 y.04.47352 T0.99 T249()27e**

2. **Solve Poisson's equation:**

$$u_{xx} + u_{yy} = \delta(x - x_0)\delta(y - y_0)$$

in the region $R = \{(x, y) | 0 < x < a,\ 0 < y < b\}$, **assuming** $(x_0, y_0) \in R$ **subject to the boundary conditions** $u(0, y) = u(a, y) = 0$, $u_y(x, 0) = u(x, b) = 0$

Give your answer in the form of a single Fourier sine series:

$$u(x, y) = \frac{2}{a} \sum_{n=1}^{\infty} A(n, y) \sin\left(\frac{n\pi x}{a}\right)$$

The function $A(n, y)$ **may be written in a piecewise manner:** $A(n, y) = \begin{cases} A_1(n, y) & , \quad 0 < y < y_0 \\ A_2(n, y) & , \quad y_0 < y < b \end{cases}$

3. **Find the solution of the boundary value problem**

$$\begin{aligned} u_t = u_{xx} - \alpha u + g(t) \quad &, \quad x, t > 0 \\ u(x, 0) = f(x) \quad &, \quad 0 \le x < \infty \\ u_x(0, t) = 0 \quad &, \quad t > 0 \end{aligned}$$

with $u < \infty$ **as** $x, t \to \infty$

Simplify the solution for the case when $f(x) = \begin{cases} 4 & , \quad 0 \le x < 2 \\ 0 & , \quad 2 \le x < \infty \end{cases}$ **,** $\alpha = \frac{1}{4}$ **and** $g(t) = 1 + e^{-\alpha t}$**.**

Plot $u(0, t)$ **for** $t \in [0, 12]$**.**