Syllabus of PhD Preliminary Examination in Numerical Analysis[1]

NA Prelim Committee
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The PhD Preliminary Examination in Numerical Analysis is intended to determine whether a student has adequate knowledge at the undergraduate level in numerical analysis including numerical linear algebra to begin a Ph.D. thesis research in applied mathematics. The exam is written and graded by a committee of professors in the Department of Mathematics with expertise in numerical analysis.

The written exam consists of approximately 6 to 8 questions. The students are given four hours to take the exam. Students are not allowed use of notes or books, but allowed use of a calculator. A passing score is 70% or higher. Students who fail the exam can take the exam one more time.

Students interested in taking the exam should have taken the courses, Math 410 { Numerical Methods and Math 411 { Numerical Linear Algebra, or their equivalents. A student should prepare for the exam by studying the relevant material from several books listed at the end of this syllabus, and by reviewing problems of the practice test and of previously given exams.

Numerical Analysis

1. General concepts

   Floating point numbers and operations, roundo errors, loss of signi cance, degree of precision of an approximation formula, Richarsdon extrapolation, the Aitken acceleration, Taylor series, Chebyshev polynomials, convergence of sequences, convergence orders, computational complexity of algorithms.

2. Interpolation

   Lagrange interpolation polynomial, divided di erences, Chebyshev points, Hermite interpolation, spline interpolation, error analysis, algorithms and costs.

3. Function and data approximation

   Continuous and discrete least squares approximations, orthogonal polynomials, Gram-Schmidt process, trigonometric least squares approximation, trigonometric interpolation, discrete Fourier transforms, fast Fourier transform algorithm.

4. Numerical di erentiation

   Finite di erence formulas, di erentiation using polynomial interpolation, error analysis and sensitivity to roundo errors.

5. Numerical integration

   Closed and open Newton-Cotes quadratures, Gaussian quadrature, composite formulas, Romberg integration, adaptive integration, error analysis, algorithms, and costs.

6. Methods for nding zeros of functions of one variable

   Bisection method, xed point iteration, Newton's method, secant method, convergence analysis of iterative methods.

7. Methods for nding zeros of functions of several variables

   Fixed point iteration, Newton's method, steepest descent method, homotopy and continuation methods.

[1] Last updated: February, 2017

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References

Main

1. R. Burden and J. Faires, Numerical Analysis, Brooks/Cole, 9 edition, 2010.
2. W. Cheney and D. Kincaid, Numerical Mathematics and Computing, Brooks/Cole, 7th edition, 2012.
3. D. Watkins, Fundamentals of Matrix Computations, John Wiley & Sons, 3rd edition, 2010.

Supplementary

1. U. Ascher and C. Greif, A First Course in Numerical Methods, SIAM, 2011.
2. G. Dahlquist and A. Bjork, Numerical Methods, Dover, 2003.
3. J. Demmel, Applied Numerical Linear Algebra, SIAM, 1997.