

Numerical Analysis Qualifying Exam  
 Mathematics Department  
 New Mexico Tech  
 Fall, 2005

(Answer all 6 questions.)

1. Given the definition of the 2-norm of a matrix

$$\|A\|_2 = \max_{\|x\|_2=1} \|Ax\|_2$$

Show that

$$\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}$$

Hint: Consider the constrained problem

$$\max \|Ax\|_2^2$$

subject to

$$\|x\|_2^2 = 1$$

Apply the Lagrange multiplier technique to this problem.

2. A finite difference formula for the first derivative is

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2} f''(c)$$

where  $c$  is some point in the interval  $(x, x+h)$ . Suppose that  $|f''(z)| \leq M$  on the interval  $(x, x+h)$  and that the values of  $f(x)$  and  $f(x+h)$  are known.

(a) The Jacobi iteration algorithm creates the sequence of approxima-