Probability and Statistics, Sample Prelim II Questions, Fall 2021

1. Suppose the random variable X has pdf

$$f(x;\ ) = \begin{cases} X^{\ 1} & \text{for } x \quad 1 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the Je rey's prior ( )

   (b) Find the MLE of and the Fisher's information.

   (c) Find the 95% CI for based on a sample of size $n = 100$, with $\prod_{i=1}^{n} X_i = 40320$.

   (d) Find the 95% CI for p_- based on Delta method.

2. Let X_k have Gamma distribution with $= k$, k is an integer, and $=$.

   (a) Show that X_k/k converges to as $k \ ! \ 1$.

   (b) Show that

$$\frac{X_k}{p} \quad k$$

(c) Compute the bias and variance of each estimator. Which estimator would you prefer and why?

4. Suppose that $X_1; X_2; : : : ; X_n$

6. Consider a Poisson process $X(t)$ with intensity $\quad$, so that

$$p_k(t) = P(X(t) = k) = \exp(\quad t)\frac{(\ t)^k}{k!}:$$

Another Poisson process $Y(t)$, independent of the rst one, has the intensity $\quad$. Show that $X(t) + Y(t)$ is also a Poisson process, and nd its intensity.

7. Let $X(t)$ be a pure death process with initial value $X(0) = N$ and the death rate $_n = n \ ; n = N; N \quad 1; :::; 1$. Let $P_n(t) = P(X(t) = n)$. Find a system of di erential equations for $P_n(t)$ and show that their solution is

$$P_n(t) = \begin{matrix} N \\ n \end{matrix} \quad e^{\ n\ t}$$

t_n .7988T1.9551 2.9552 Tf 13.789 -25.058 Td [8 (