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Name: Sharmin Sultana

**PhD Preliminary Examination in Analysis**  
Department of Mathematics  
New Mexico Tech  
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1. Let  $\{x_n\}_{n=1}^{\infty}$  be a sequence of real numbers given by

$$x_1 > \frac{\rho-}{2} \quad \text{and} \quad x_{n+1} = \frac{2 + x_n}{1 + x_n}; \quad n = 1; 2; \dots$$

Prove that the sequence  $\{x_n\}_{n=1}^{\infty}$  is convergent and find its limit.

2. Let  $f : [0;1] \rightarrow \mathbb{R}$  and  $g : [0;1] \rightarrow \mathbb{R}$  be two real valued functions on the interval  $[0;1]$ . Suppose that the function  $f$  is monotone and the function  $g$  is increasing and continuous. Prove that  $f \circ g$  is

7. Let  $P$  be a polynomial of degree  $n$ . Let  $C$  be a sufficiently large circle centered at the origin and oriented counterclockwise. Show that

$$I = \int_C \frac{dz}{2\pi i} \frac{P'(z)}{P(z)} = n$$

8. Let  $\alpha, \beta \in \mathbb{R}$  be two real numbers. Use principal value integrals to evaluate  $\int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{x + i\beta} dx$ .